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446. Proposed by S. G. BARTON, University of Pennsylvania.

Prove any one or more of the sixteen theorems, stated without proof, in the article in this issue of the MONTHLY (pages 182-184) on "Properties of the Normals to a Conic."

CALCULUS.

366. Proposed by I. A. BARNETT, University of Chicago.

Compute the definite integral $\int_a^b \sin^{-1} x dx$ where $0 \leq a \leq 1$ and $0 \leq b \leq 1$, by direct summation.

367. Proposed by C. N. SCHMALL, New York City.

Show that the volume enclosed by the surface $(x^2 + y^2 + z^2)^5 = (a^3x^2 + b^3y^2 + c^3z^2)^2$ is $\frac{4}{3}\pi(a^3 + b^3 + c^3)$.

MECHANICS.

295. Proposed by B. F. FINKEL, Drury College.

A homogeneous hollow cylinder, whose inner radius is half of its outer radius, rolls without slipping down a plane inclined at an angle α to the horizontal. Find its acceleration.

[From Prescott's *Mechanics of Particles and of Rigid Bodies*.]

NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, Princeton, N. J.

If p is prime and > 3 , show that $\sum_{a=1}^{a=p-1} 1/a^2 \equiv 0 \pmod{p}$.

219. Proposed by R. D. CARMICHAEL, Indiana University.

Determine whether it is possible for a polygon to have the number of its diagonals equal to a perfect fourth power.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

A solution of 399 by WM. CULLUM and a solution of 400 by LOUIS O'SHAUGHNESSY were received too late for credit in the May issue.

401. Proposed by R. D. CARMICHAEL, Indiana University.

Prove the validity of Borda's series:

$$\log(x+2) = 2 \log(x+1) - 2 \log(x-1) + \log(x-2) \\ + 2 \left[\frac{2}{x^3-3x} + \frac{1}{3} \left(\frac{2}{x^3-3x} \right)^3 + \frac{1}{5} \left(\frac{2}{x^3-3x} \right)^5 + \cdots \right].$$

SOLUTION BY A. M. HARDING, University of Arkansas.

We have

$$\log(x+2) - 2 \log(x+1) + 2 \log(x-1) - \log(x-2) \\ = \log \frac{(x-1)^2(x+2)}{(x+1)^2(x-2)} = \log \frac{x^3-3x+2}{x^3-3x-2} = \log \frac{1 + \frac{2}{x^3-3x}}{1 - \frac{2}{x^3-3x}} \\ = 2 \left[\frac{2}{x^3-3x} + \frac{1}{3} \left(\frac{2}{x^3-3x} \right)^3 + \frac{1}{5} \left(\frac{2}{x^3-3x} \right)^5 + \cdots \right],$$

which proves the result.

It should be observed that the series converges only for values of x such that $x^3 - 3x > 2$. But $x^3 - 3x + 2 = (x + 1)^2(x - 2)$. Hence x must be greater than 2.

Also solved by A. G. CARIS, F. M. MORGAN, HORACE OLSON, ELIJAH SWIFT, S. W. REAVES, and T. M. BLAKESLEE.

402. Proposed by R. D. CARMICHAEL, Indiana University.

Obtain other series similar to that of Borda given in the preceding problem (No. 401).

SOLUTION BY S. W. REAVES, University of Oklahoma.

If in the well-known series

$$(1) \quad \log(n+1) = \log n + 2 \left[\frac{1}{2n+1} + \frac{1}{3} \left(\frac{1}{2n+1} \right)^3 + \frac{1}{5} \left(\frac{1}{2n+1} \right)^5 + \cdots \right]$$

we set

$$n = \frac{(x-2k)(x+k)^2}{4k^3},$$

and therefore

$$n+1 = \frac{(x+2k)(x-k)^2}{4k^3}, \quad \text{and} \quad \frac{1}{2n+1} = \frac{2k^3}{x^3 - 3k^2x},$$

we shall have a generalized Borda's series

$$(2) \quad \begin{aligned} \log(x+2k) &= 2 \log(x+k) - 2 \log(x-k) + \log(x-2k) \\ &+ 2 \left[\frac{2k^3}{x^3 - 3k^2x} + \frac{1}{3} \left(\frac{2k^3}{x^3 - 3k^2x} \right)^3 + \frac{1}{5} \left(\frac{2k^3}{x^3 - 3k^2x} \right)^5 + \cdots \right]. \end{aligned}$$

For $k = 1$, (2) becomes Borda's series.

Again, if in (1) we set

$$n = \frac{(x-3k)(x+k)^3}{16k^3x},$$

we shall have

$$(3) \quad \begin{aligned} \log(x+3k) &= 3 \log(x+k) - 3 \log(x-k) + \log(x-3k) \\ &+ 2 \left[\frac{8k^3x}{x^4 - 6k^2x^2 - 3k^4} + \frac{1}{3} \left(\frac{8k^3x}{x^4 - 6k^2x^2 - 3k^4} \right)^3 + \cdots \right]. \end{aligned}$$

Also solved by F. M. MORGAN, T. M. BLAKESLEE, and A. M. HARDING.

GEOMETRY.

410. Proposed by A. H. HOLMES, Brunswick, Maine.

Given a focus and two tangents to an ellipse, prove that the locus of the foot of the normal corresponding to either tangent is a straight line.

SOLUTION BY A. M. HARDING, University of Arkansas.

Let SP and SP' be the fixed tangents, F the fixed focus, F' the other focus, and N the foot of the normal corresponding to SP . Draw FE and $F'E'$ perpendicular to SP , and FG and $F'G'$ perpendicular to SP' . Draw FF' cutting SP in L .